2020/TDC/ODD/SEM/MTMH–101/254

TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Honours)

(1st Semester)

Course No. : MTMH–101

(Classical Algebra and Trigonometry)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

GROUP—A

(Classical Algebra)

(*Marks* : 30)

UNIT—I

1. *(a)* Prove that the matrix A^2 is symmetric only if either A is symmetric or A is skew-symmetric. 3

(b) Find the rank of the matrix

$$A \quad \begin{matrix} 4 & 2 & 1 & 3 \\ 1 & 0 & 3 & 9 \\ 2 & 1 & 3 & 4 \end{matrix}$$

by reducing it to Echelon form. 3

(c) Solve the following system of equations :

$$\begin{matrix} x \quad y \quad z & 8 \\ x \quad y \quad 2z & 6 \\ 3x \quad 5y \quad 7z & 14 \end{matrix}$$

4

2. *(a)* Reduce the matrix

$$A \quad \begin{matrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 3 & 2 & 1 & 0 \end{matrix}$$

to normal form and hence find its rank. 4

(b) Check the following system of equations for consistency : 2

$$\begin{matrix} x \quad y \quad 3z & 2 \\ 5x \quad y \quad 8z & 9 \\ 3x \quad 3y \quad 14z & 7 \end{matrix}$$

(c) If A is a square matrix of order n, show that

$$A.(\text{adj}A) \quad (\text{adj}A) \quad A \quad |A|I$$

and hence deduce that $|\text{adj}A| \quad |A|^{n \quad 1}$ for a non-singular matrix. 4

UNIT—II

3. *(a)* Find the condition that the roots of the equation

$$x^3 \quad px^2 \quad p^2x \quad r \quad 0$$

are in AP. 3

(b) Solve $x^3 \quad 6x \quad 4 \quad 0$ by Cardan's method. 4

(c) Prove the AM-GM inequality. 3

4. *(a)* If , , are the roots of the equation

$$x^3 \quad x^2 \quad x \quad 5 \quad 0$$

then form the equation whose roots are

$$-, \quad \frac{1}{} \text{ and } \quad \frac{1}{}.$$ 3

(b) Find the biquadratic equation with real coefficients, two of whose roots are $2i \quad 1$. 3

(c) If $a, b, c, d \quad 0$ and $abcd \quad 1$, then find the minimum possible value of $(1 \quad a)(1 \quad b)(1 \quad c)(1 \quad d)$. 4

UNIT—III

5. *(a)* Show that every convergent sequence is bounded. Give example of a bounded sequence that is not convergent. 3+1=4

(b) Test the convergence of the series

$$1 \quad \frac{3}{2} \quad \frac{5}{3} \quad \frac{7}{4} \quad \cdots$$ 3

(c) Test the convergence of

$$1 \quad \frac{x}{2} \quad \frac{x^2}{3^2} \quad \frac{x^3}{4^3} \quad \cdots, x \quad 0$$ 3

6. *(a)* Prove that every monotonic increasing sequence that is bounded above is convergent. 4

(b) Test the convergence of the series

$$\frac{x}{1} \quad \frac{1}{2}\frac{x^2}{3} \quad \frac{1 \quad 3}{2 \quad 4}\frac{x^3}{5} \quad \frac{1 \quad 3 \quad 5}{2 \quad 4 \quad 6}\frac{x^4}{7} \quad \cdots (x \quad 0)$$ 4

(c) Define Cauchy sequence. Give an example. 2

GROUP—B

(Trigonometry)

(*Marks* : 20)

UNIT—IV

7. *(a)* Solve the equation

$$1 \quad x \quad x^2 \quad \cdots \quad x^6 \quad 0$$

using De-Moivre's theorem. 3

(b) Prove that

$$\frac{\sin^3}{3!} \quad \frac{3}{3!} \quad (1 \quad 3^2)\frac{5}{5!} \quad (1 \quad 3^2 \quad 3^4)\frac{7}{7!} \quad \cdots$$ 4

(c) If

$$(1 \quad x)^n \quad p_0 \quad p_1 x \quad p_2 x^2 \quad p_3 x^3 \quad \cdots$$

show that

$$p_0 \quad p_2 \quad p_4 \quad \cdots \quad 2^{n/2} \cos\frac{n}{4}$$

$$p_1 \quad p_3 \quad p_5 \quad \cdots \quad 2^{n/2} \sin\frac{n}{4}$$ 3

8. *(a)* State and prove De-Moivre's theorem for integral index. 1+4=5

(b) Prove that

$$\frac{1 \quad \sin \quad i\cos}{1 \quad \sin \quad i\cos}^{n} \quad \cos(\frac{n}{2} \quad n\,) \quad i\sin(\frac{n}{2} \quad n\,)$$

$$(\sin \quad i\cos\,)^n$$

1+4=5

UNIT—V

9. *(a)* Separate into real and imaginary parts

$$\tan^{\ 1}(x \quad iy)$$ 4

(b) Prove that

$$\frac{}{8} \quad \frac{1}{1\ 3} \quad \frac{1}{5\ 7} \quad \frac{1}{9\ 11} \quad \cdots$$ 3

(c) Sum to n terms

$$\cos \quad \cos 2 \quad \cos 3 \quad \cos 4$$
$$\cos 5 \quad \cos 6 \quad \cdots$$ 3

10. *(a)* If $\sin x \quad n\sin(x \quad)$, expand x in a series of ascending power of n. 4

(b) Find the sum

$$\sin \quad \frac{}{2}\sin 2 \quad \frac{}{3}\sin 3 \quad \cdots$$
$$(\qquad)$$ 4

(c) Show that

$$\cos h^2 \quad \sin h^2 \quad 1$$ 2

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