2020/TDC/ODD/SEM/MTMH–103/256

TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Honours)

(1st Semester)

Course No. : MTMH–103

(Geometry)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Find the angle through which the axes are to be rotated so that the equation $\sqrt{3}x \quad y \quad 6 \quad 0$ may be reduced to the form $x \quad c$. Also determine the value of c. 3

(b) Find the angle through which the axes are to be rotated so that the equation $ax^2 \quad 2hxy \quad by^2 \quad 0$ becomes another equation in which the term 'xy' is absent. 3

(c) Find the condition that general equation of second degree represents a pair of straight lines. 4

2. *(a)* Show that the area of the triangle formed by the straight lines $ax^2 \quad 2hxy \quad by^2 \quad 0$ and $lx \quad my \quad 1$ is

$$\frac{\sqrt{h^2 \quad ab}}{am^2 \quad 2hlm \quad bl^2}$$ 5

(b) Reduce the equation

$$3x^2 \quad 2xy \quad 3y^2 \quad 16x \quad 20 \quad 0$$

to standard form and determine its nature. 5

UNIT—II

3. *(a)* Find the coordinate of the pole of the straight line $x \quad 8y \quad 12$ with respect to the conic $\frac{x^2}{36} \quad \frac{y^2}{9} \quad 1$. 3

(b) Prove that the straight line $lx \quad my \quad n \quad 0$ is a normal to the ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$, if $\frac{a^2}{l^2} \quad \frac{b^2}{m^2} \quad \frac{(a^2 \quad b^2)^2}{n^2}$. 4

(c) Find the equation of the diameter of the ellipse $3x^2 \quad 4y^2 \quad 5$ conjugate to the diameter $y \quad 3x \quad 0$. 3

4. *(a)* The polar of the point Q with respect to the circle $x^2 \quad y^2 \quad a^2$ touches the circle $4x^2 \quad 4y^2 \quad a^2$. Show that the locus of Q is the circle $x^2 \quad y^2 \quad 4a^2$. 3

(b) If the normal to the hyperbola $xy \quad c^2$ at the point $(ct_1, c/t_1)$ meets the curve again at the point $(ct_2, c/t_2)$, then show that $t_1^3 t_2 \quad 1 \quad 0$. 4

(c) For the hyperbola $16x^2 \quad 9y^2 \quad 144$, find the equation of the diameter which is conjugate to the diameter whose equation is $x \quad 2y$. 3

UNIT—III

5. *(a)* Prove that in a conic, the semi-latus rectum is the harmonic mean between the segments of a focal chord. 4

(b) A chord PQ of a conic whose eccentricity is e and semi-latus rectum l subtends a right angle at the focus S, show that

$$\frac{1}{SP} \quad \frac{1}{l}^{\;2} \quad \frac{1}{SQ} \quad \frac{1}{l}^{\;2} \quad \frac{e^2}{l^2}.$$

4

(c) If e and e be the eccentricities of a hyperbola and its conjugate, then show that $\frac{1}{e^2} \quad \frac{1}{e^{\;2}} \quad 1.$ 2

6. *(a)* Find the polar equation of a conic with latus rectum of length $2l$, eccentricity e and the focus being the pole. 5

(b) If PSQ and $PS\,R$ be two chords of an ellipse through focus S and S, show that $\frac{PS}{SQ} \quad \frac{PS}{S\,R}$ is independent of the position of P. 5

UNIT—IV

7. *(a)* Prove that the straight lines

$$\frac{x \quad 1}{2} \quad \frac{y \quad 1}{3} \quad \frac{z \quad 10}{8} \text{ and }$$

$$\frac{x \quad 4}{1} \quad \frac{y \quad 3}{4} \quad \frac{z \quad 1}{7}$$

intersect and find the plane through them. Also find their point of intersection. 5

(b) A plane passing through a fixed point (a, b, c) cuts the axes in A, B, C. Show that the locus of the centre of the sphere $OABC$ is $\frac{a}{x} \quad \frac{b}{y} \quad \frac{c}{z} \quad 2$. 5

8. *(a)* Show that $2x \quad 6y \quad 3z \quad 49 \quad 0$ is a tangent plane to the sphere $x^2 \quad y^2 \quad z^2 \quad 49$. Also find the point of contact. 2+2=4

(b) Find the equation of the sphere through the points (0, 0, 0), (0, 1, 1), (1, 2, 0) and (1, 2, 3). 4

(c) What is the equation of a tangent plane to the sphere

$$x^2 \quad y^2 \quad z^2 \quad 2x \quad 2y \quad 2z \quad 6 \quad 0$$

at the point (6, –3, –2)? 2

UNIT—V

9. (a) Define a cone and guiding curve. 2

(b) Find the equation of right circular cone whose vertex is the point (1, –2, –1), semi-vertical angle 60° and axis is

$$\frac{x \quad 1}{3} \quad \frac{y \quad 2}{4} \quad \frac{z \quad 1}{5}$$ 4

(c) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{1} \quad \frac{y}{2} \quad \frac{z}{3}$ and whose guiding curve is $x^2 \quad y^2 \quad 9, \; z \quad 1$. 4

10. (a) Define a cylinder and generator of a cylinder. 2

(b) Prove that $lx \quad my \quad nz \quad p$ is a tangent plane to the conicoid $ax^2 \quad by^2 \quad cz^2 \quad 1$, if $\frac{l^2}{a} \quad \frac{m^2}{b} \quad \frac{n^2}{c} \quad p^2$. 4

(c) Find the equation of the cone whose vertex is the origin and which passes through the curve of intersection of the plane $lx \quad my \quad nz \quad p$ and the surface $ax^2 \quad by^2 \quad cz^2 \quad 1$. 4

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