2020/TDC/ODD/SEM/MTMH–301/257

TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Honours)

(3rd Semester)

Course No. : MTMH–301

(Real Analysis)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* State and prove Young's theorem. 1+4=5

(b) If $x \quad u \quad v, y \quad uv$ and z is a function of x and y, prove that

$$\frac{{}^2 z}{u^2} \quad 2\frac{{}^2 z}{u \ \ v} \quad \frac{{}^2 z}{v^2} \quad (x^2 \quad 4y)\frac{{}^2 z}{y^2} \quad 2\frac{z}{y}$$ 5

2. *(a)* Show that the function

$$f(x, y) \quad \frac{xy(x^2 \quad y^2)}{x^2 \quad y^2}, \quad (x, y) \quad (0, 0)$$

$$f(0, 0) \quad 0$$

does not satisfy the conditions of Schwarz's theorem and $f_{xy}(0, 0) \quad f_{yx}(0, 0)$. 5

(b) If $x \quad f(r, s),\quad y \quad g(r, s)$ and $J \quad \frac{(f,\ g)}{(r,\ s)} \quad 0$, if h is a function of r, s, show that

$$\frac{h}{x} \quad \frac{1}{J}\frac{(h,\ g)}{(r,\ s)}, \quad \frac{h}{y} \quad \frac{1}{J}\frac{(f,\ g)}{(r,\ s)}$$ 5

UNIT—II

3. *(a)* Discuss maxima or minima of

$$f(x, y) \quad 2\sin\frac{x \quad y}{2}\cos\frac{x \quad y}{2} \quad \cos(x \quad y),\ x, y \quad 0$$ 5

(b) Apply Lagrange's method of undetermined multipliers to find the maximum value of $u \quad (xyz)^2$, when $x^2 \quad y^2 \quad z^2 \quad 1$. 5

4. *(a)* Describe Lagrange's method of undetermined multipliers for determining the stationary values of a function of several variables. 5

(b) A rectangular box, open at the top is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction. 5

UNIT—III

5. (a) Show that if $f:[a, b]\to\mathbb{R}$ is continuous, then f is Riemann integrable on $[a, b]$. 4

(b) If a function f is bounded and integrable on $[a, b]$, then prove that the function F defined as

$$F(x)=\int_a^x f(t)dt,\ a\le x\le b$$

is continuous on $[a, b]$ and further more if f is continuous at a point c of $[a, b]$, then F is derivable at c and $F'(c)=f(c)$. 6

6. (a) Show that the function $[x]$, where $[x]$ denotes the greatest integer not greater than x, is integrable on $[0, 3]$ and $\int_0^3 [x]dx = 3$. 3

(b) f is a non-negative continuous function on $[a, b]$ and $\int_a^b f dx = 0$. Prove that $f(x)=0$ for all $x\in[a, b]$. 4

(c) If $f:[a, b]\to\mathbb{R}$ be bounded and R-integrable on $[a, b]$, then prove that $|f|$ is bounded and R-integrable on $[a, b]$. 3

UNIT—IV

7. (a) For what values of m and n is the integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}\log x\,dx$$

convergent? 5

(b) Establish the Lagrange's duplication formula

$$\sqrt{\pi}\,\Gamma(2m)=2^{2m-1}\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)$$ 5

8. (a) Using differentiation under integral sign, show that

$$\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}dx=\frac{\pi}{2}\log(1+a)$$ 5

(b) Show that

$$\int_0^1 x^{m-1}(1-x)^{n-1}dx$$

exists if and only if m, n are both positive. 5

UNIT—V

9. *(a)* State and prove Stokes' theorem. 5

(b) Evaluate the surface integral

$$\quad_{S} \; x^3 dydz \quad y^3 dzdx \quad z^3 dxdy$$

where S is the outer surface of the sphere $x^2 \quad y^2 \quad z^2 \quad 1$. 5

10. *(a)* Prove that the line integral

$$(yx^3 \quad xe^y)dx \quad (xy^3 \quad ye^y \quad 2y)dy$$

equals zero, if is a closed curve symmetrical with respect to the origin. 5

(b) Evaluate

$$\quad_{R} \; [x \quad y]dxdy$$

over the rectangle $R \quad [0, 1; 0, 2]$; where $[x \quad y]$ denotes the greatest integer less than or equal to $(x \quad y)$. 5

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