TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Honours)

(3rd Semester)

Course No. : MTMH–302

(Statics)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove that any system of coplanar forces acting on a rigid body can be reduced ultimately to either a single force or a single couple, unless it is in equilibrium. 5

(b) Under what conditions, three coplanar forces acting on a body be in equilibrium?

A heavy uniform rod of length $2a$ rests in equilibrium having one end against a smooth vertical wall and being placed upon a peg at a distance b from the wall. Prove that the inclination of the rod to the horizontal is

$$\cos^{\ 1}\ \frac{b}{a}\ ^{1/3}$$

1+4=5

2. *(a)* Forces P, Q, R, S act along the sides

$$AB,\ BC,\ CD,\ DA$$

respectively of the cyclic quadrilateral $ABCD$, where A and B are the extremities of a diameter. If they are in equilibrium, prove that

$$R^2 \quad P^2 \quad Q^2 \quad S^2 \quad \frac{2PQS}{R}$$ 5

(b) A uniform ladder rests in limiting equilibrium with one end against a rough vertical wall and other on a rough horizontal plane, the angles of friction being and respectively. Show that the inclination of the ladder to the horizon is given by

$$\tan \quad \frac{\cos(\qquad)}{2\sin \quad \cos}$$ 5

UNIT—II

3. *(a)* Find the CG of a uniform sector of a circle. 5

(b) Find the CG of the arc of the astroid $x^{2/3} \quad y^{2/3} \quad a^{2/3}$ lying in the 1st quadrant. 5

4. *(a)* Find the centre of gravity of a solid uniform hemisphere of radius *a*. 5

(b) Draw a neat diagram of the second system of pulleys. Derive the relation between effort and weight. 3+2=5

UNIT—III

5. *(a)* Enumerate the nature of forces which may be omitted in forming the equation of virtual work. 5

(b) A regular hexagon is composed of six equal heavy rods freely jointed together and two opposite angles are connected by a string which is horizontal. One rod being in contact with the horizontal table, at the middle point of the opposite rod is placed a weight W_1. If W be the weight of each rod, show that the tension of the string is $\frac{3W \quad W_1}{\sqrt{3}}$. 5

6. *(a)* A sting of length *a* forms the shorter diagonal of a rhombus of four uniform rods, each of length *b* and weight *w* which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension in the string is

$$\frac{2w(2b^2 \quad a^2)}{b\sqrt{4b^2 \quad a^2}}$$ 5

(b) A heavy elastic string, whose natural length is 2 *a*, is placed round a smooth cone whose axis is vertical and whose semi-vertical angle is . If *w* be the weight and be the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle of radius $a(1 \quad \frac{w}{2} \cot \quad)$. 5

UNIT—IV

7. *(a)* A hemisphere rests in equilibrium on a sphere of equal radius. Show that the equilibrium is unstable when the curved and stable when the flat surface of the hemisphere rests on the sphere. 5

(b) A heavy uniform rod of length $2a$ rests partly inside and partly outside a fixed smooth hemispherical bowl of radius r, the rim of the bowl is horizontal and one point of the rod is in contact with the rim. If be the inclination of the rod to the horizon, show that $2r\cos 2 \quad a\cos$. Show also that equilibrium is stable. 5

8. *(a)* A body consisting of a cone and hemisphere on the same base rests on a rough horizontal table, the hemisphere being in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}$ times the radius of the hemisphere. 5

(b) Show that for a system in equilibrium, under conservative forces, the potential energy is minimum for stable and maximum for unstable equilibrium. 5

UNIT—V

9. *(a)* Define a common catenary and obtain the Cartesian equation in the form of $y \quad c\ \cosh(x/c)$. 5

(b) A telegraph wire is made of a given material and such a length l is stretched between two posts, distance d apart and at the same height, as will produce the least possible tension at the posts. Show that

$$l \quad \frac{d}{\;}\sinh \quad , \text{ where}$$

is given by the equation

$$\tanh \quad 1$$ 5

10. *(a)* Deduce the equations

$$mX \quad \frac{d}{ds} \quad T\frac{dx}{ds} \quad 0$$

$$mY \quad \frac{d}{ds} \quad T\frac{dy}{ds} \quad 0$$

for equilibrium of strings under given forces. 5

(b) If T is the tension at any point P of a catenary and T_0 that of the lowest point C, then prove that $T^2 \quad T_0^2 \quad W^2$, where W is the weight of the arc CP of the catenary. 5

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