TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Honours)

(3rd Semester)

Course No. : MTMH–303

(Linear Algebra)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Show that the necessary and sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a subspace of V is a, b F and , W a b W. 5

(b) Show that the vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) are linearly independent in $\mathbb{R}^3$. 5

2. *(a)* If a vector space V is the set of all real valued continuous functions over the field of real numbers $\mathbb{R}$, then show that the set W of solutions of the differential equation

$$2\frac{d^2y}{dx^2} \quad 9\frac{dy}{dx} \quad 2y \quad 0$$

is a subspace of V. 5

(b) Show that the set S $\{1, x, x^2, ..., x^n\}$ of $(n$ $1)$ polynomials in x is a basis of the vector space $P_n(\mathbb{R})$ of all polynomials in x of degree n over the field of real numbers. 5

UNIT—II

3. *(a)* Let U and V be two vector spaces over the same field F and let T be a linear transformation from U into V. If U is finite dimensional, prove that

$$\text{rank }(T) \quad \text{nullity }(T) \quad \dim U$$ 4

(b) Consider the basis $\{ \ _1, \ _2, \ _3\}$ of $\mathbb{R}^3$, where $_1$ (1, 1, 1), $_2$ (1, 1, 0) and $_3$ (1, 0, 0). Let $T : \mathbb{R}^3$ $\mathbb{R}^2$ be a linear transformation defined as $T(\ _1)$ (1, 0), $T(\ _2)$ (2, 1) and $T(\ _3)$ (4, 3). Find $T(2, \ 3, 5)$. 4

(c) Find the rank of the linear transformation $T:\mathbb{R}^2 \quad \mathbb{R}^3$ defined by

$$T(x, y) \quad (x \quad y, x \quad y, y)$$ 2

4. (a) Find the matrix form of the linear transformation $T:\mathbb{R}^3 \quad \mathbb{R}^3$ given by

$$T(x, y, z) \quad (2y \quad z, x \quad 4y, 3x)$$

with respect to the ordered basis

$$\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$$ 5

(b) Let T be a linear operator on $\mathbb{R}^3$ defined by

$$T(x, y, z) \quad (3x, x \quad y, 2x \quad y \quad z)$$

$$(x, y, z) \quad \mathbb{R}^3$$

Is T invertible? If so, find $T^{\ 1}$. 5

UNIT—III

5. (a) Reduce the matrix

$$\begin{matrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{matrix}$$

into row-reduced Echelon form and hence find its rank. 5

(b) Prove that the matrix

$$\begin{matrix} \frac{1 \quad i}{2} & \frac{1 \quad i}{2} \\ \frac{1 \quad i}{2} & \frac{1 \quad i}{2} \end{matrix}$$

is unitary. 5

6. (a) Solve completely the system of equations

$$\begin{matrix} x \quad 3y \quad 2z \quad 0 \\ 2x \quad y \quad 4z \quad 0 \\ x \quad 11y \quad 14z \quad 0 \end{matrix}$$ 5

(b) Show that the system of equations

$$\begin{matrix} x \quad 2y \quad z \quad 3 \\ 3x \quad y \quad 2z \quad 1 \\ 2x \quad 2y \quad 3z \quad 2 \\ x \quad y \quad z \quad 1 \end{matrix}$$

is consistent and solve them. 5

UNIT—IV

7. (a) Determine the eigenvalues and eigenvectors of the matrix

$$\begin{matrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{matrix}$$ 5

(b) Show that the matrices A and A have the same eigenvalues. 3

(c) Show that 0 is an eigenvalue of a matrix if and only if the matrix is singular. 2

8. *(a)* If A is a non-singular matrix, then prove that the eigenvalues of $A^{\ \ 1}$ are the reciprocals of the eigenvalues of A. 5

(b) Find the eigenvalues and eigenvectors of the linear transformation $T : \mathbb{R}^2 \quad \mathbb{R}^2$ defined by $T(x, y) \quad (\ x, \quad y)$. 5

UNIT—V

9. *(a)* Verify Cayley-Hamilton theorem for the matrix

$$\begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix}$$

and hence find its inverse. 5

(b) If and are vectors in an inner product space, then show that

$$\| \quad \|^2 \quad \| \quad \|^2 \quad 2\{\| \quad \|^2 \quad \| \quad \|^2\}$$ 5

10. *(a)* Find the dual basis of the basis set

$$B \quad \{(1, \quad 1), (0, 1)\} \text{ for } \mathbb{R}^2$$ 5

(b) If and are two vectors in a real inner product space and if $\| \quad \| \quad \| \quad \|$, then show that and are orthogonal. Interpret the result geometrically. 5

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