2020/TDC/ODD/SEM/
MTMH–503 (A/B)/262

TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Honours)

(5th Semester)

Course No. : MTMH–503

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* Option—A *or* Option—B

OPTION—A

Course No. : MTMH–503 (A)

(ADVANCED ALGEBRA)

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Let G be a group. Show that the set of conjugate classes of G is a partition of G. 5

(b) Let G be a finite group of order p^n, where p is a prime and n 0. Then show that G has a non-trivial centre. 5

2. *(a)* Let G be a group containing an element of finite order n 1 and exactly two conjugacy classes. Prove that $|G|$ 2. 5

(b) Define conjugacy class in a group. List all the conjugacy classes of S_3. 5

UNIT—II

3. *(a)* Locate the system normalizers of S_4. 5

(b) Let H be a subgroup of a group G. Prove that the number of conjugates of H in G is the index of $N(H)$ in G. 5

4. *(a)* Let H be a proper subgroup of finite order group G. Show that G is not the union of all conjugates of H. 5

(b) Determine the class equation for non-Abelian groups of order 39 and 55. 5

UNIT—III

5. *(a)* Let G and H be finite cyclic groups. Show that G H is cyclic iff $|G|$ and $|H|$ are relatively prime. 5

(b) Prove that a factor group of a solvable group is solvable. 5

6. *(a)* Prove that the external direct product of any finite number of groups is a group. 5

(b) Let N be a normal subgroup of a group G. If both N and G/N are solvable, then prove that G is solvable. 5

UNIT—IV

7. *(a)* Prove that in a PID, an element is irreducible iff it is a prime. 4

(b) Prove that every PID is a UFD. 6

8. *(a)* Show that if D is an integral domain, then $D[x]$ is an integral domain. 5

(b) If R is a commutative ring, then show that the characteristic of $R[x]$ is the same as the characteristic of R. 5

UNIT—V

9. *(a)* Show that the field of quotients of $\mathbb{Z}$ is Q. 5

(b) Prove that an R-module M is Noetherian iff every submodule of M is finitely generated. 5

10. *(a)* Let R be a ring. Prove that R is Artinian iff every non-empty set S of left ideals of R has a minimal element. 5

(b) Let D be an integral domain and F be the field of quotients of D. Show that if E is any field that contains D, then E contains a subfield that is ring-isomorphic to F. 5

OPTION—B

Course No. : MTMH–503 (B)

(SPECIAL FUNCTIONS)

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. Define Legendre's differential equation and hence find the solutions of Legendre's differential equation. 2+8=10

2. *(a)* Prove that

$$P_n(x) \quad \frac{1}{\ } \; {}_0 \frac{d}{[x \quad \sqrt{x^2 \quad 1}\cos \quad]^{n \quad 1}}$$

when n is a positive integer. Hence show that $P_{(n \quad 1)} \quad P_n$. 5+2=7

(b) Define Legendre's function of first kind. 3

UNIT—II

3. *(a)* Prove that

$$\int_{1}^{1} (x^2 \quad 1)\, P_{n \quad 1}\, P_n\, dx \quad \frac{2n(n \quad 1)}{(2n \quad 1)(2n \quad 3)}$$ 5

(b) Prove that

$$2\, J_n(x) \quad J_{n \quad 1}(x) \quad J_{n \quad 1}(x)$$

where dashes denote the differentiation with respect to x. 5

4. *(a)* Prove that $P_n(0) \quad 0$ for n odd and

$$P_n(0) \quad \frac{(\quad 1)^{n/2} \lfloor n}{2^n \{ \lfloor n/2 \}^2}$$

for n even. 3+4=7

(b) Prove that $^{n}P_n \quad {}^{x}P_n \quad P_{n \quad 1}$, where dashes denote differentiation with respect to x. 3

UNIT—III

5. *(a)* Find $L\{F(t)\}$, where

$$F(t) \quad \begin{matrix} e^t, & 0 \quad t \quad 1 \\ t, & \quad t \quad 1 \end{matrix}$$ 4

(b) Find the value of—

(i) $L^{\quad 1} \dfrac{1}{p^2 \quad 8p \quad 16}$;

(ii) $L(e^t \cos^2 t)$. 3+3=6

6. *(a)* Find $L\{F(t)\}$, where

$$F(t) \quad \begin{matrix} 1, & 0 \quad t \quad 2 \\ t, & \quad t \quad 2 \end{matrix}$$ 4

(b) Show that

$$L^{\quad 1} \frac{3s \quad 1}{s^2 \quad 2s \quad 5} \quad e^{\quad x}(3\cos 2x \quad \sin 2x)$$ 5

(c) Define Laplace transform of the function $F(t)$. 1

UNIT—IV

7. *(a)* Solve $y \quad y \quad 2e^t$, given that $y(0) \quad 2 \quad y(0)$. 5

(b) Solve

$$\frac{d^2x}{dt^2} \quad 2\frac{dx}{dt} \quad 2x \quad 0$$

when $x(0) \quad 0,\ x(0) \quad 1$. 5

8. *(a)* Solve the initial value problem

$$\frac{dy_1}{dx} \quad y_2\ ; \ \frac{dy_2}{dx} \quad y_1$$

subject to $y_1(0) \quad 1;\ y_2(0) \quad 0.$ 5

(b) Solve $Dx \quad Dy \quad t;\ D^2 x \quad y \quad e^{\ \ t},\ x(0) \quad 3,$ $x\ (0) \quad\ 2,\ y(0) \quad 0.$ 5

UNIT—V

9. *(a)* Evaluate the Fourier transform of the function

$$f(x) \quad \begin{matrix} x^2, & 0 \quad x \quad a \\ 0, & \quad x \quad a \end{matrix}$$

5

(b) Define Fourier integral of a function $f(x)$. 2

(c) Show that

$$F_s[xf(x)] \quad \frac{d}{ds}\{F_c(s)\} \quad \text{and}$$

$$F_c[xf(x)] \quad \frac{d}{ds}\{F_s(s)\}$$

1½+1½=3

10. *(a)* Evaluate Fourier transform of

$$f(x) \quad \begin{matrix} x, & |x| \quad a \\ 0, & |x| \quad a \end{matrix}$$

5

(b) Show that

$$_0 \ \frac{\cos \quad x}{^2 \quad 1} d \quad \overline{\ \ 2} e^{\ \ x}, \quad x \quad 0$$

5

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